


**PhD Thesis**

# Pure Equilibria: Existence and Inefficiency & Online Auction

**NGUYEN KIM Thang**

*Advisor:* **Christoph DÜRR**

*Committee:* **Evripidis BAMPIS** (*University of Evry, France*)
**Olivier BOURNEZ** (*Ecole Polytechnique, France*)
**Claire MATHIEU** (*Brown University, USA*)
**Peter Bro MILTERSEN** (*University of Aarhus, Denmark*)
**Jérôme RENAULT** (*Ecole Polytechnique, France*)
**Michel de ROUGEMONT** (*University of Paris 11, France*)

# PURE EQUILIBRIA: EXISTENCE AND INEFFICIENCY & ONLINE AUCTION

Nguyen Kim Thang

Ecole Polytechnique, 2009.

Every atom in universe exists in equilibrium of different physical forces. Everyone in society lives in equilibrium of their work and family, of their enthusiasm and reality. Equilibrium is one of the main notions in Game Theory — the domain studies behaviors of entities according to their own interests. The conflicting interests, the lack of coordination and regulation may not lead a game to a pure equilibrium; even if pure equilibria exist, the result of local optimization of rational players in general does not have any type of global property.

Our contributions in equilibria are twofold: we study the existence of a pure Nash equilibrium and analyze the inefficiency of equilibria in games. To prove the existence of equilibria, we use extensively the potential argument in which we figure out potential functions according to different dynamics in games. Moreover, for games which does not necessarily admit a pure equilibrium, we present an useful technique in settling the complexity of deciding whether the games possess an equilibrium.

To quantify the loss caused by selfish behaviors regarding to a social objective function of a game, we study the two well-known measures: the price of anarchy which is defined as the worst-case ratio between the social objective value of an equilibrium and the optimum; the price of stability which is defined as the worst-case ratio between the best social objective value among all equilibria and the optimum. Moreover, we introduce and analyze the social cost discrepancy — the worst-case ratio between the social objective values of two equilibria — which compares the quality of different outcomes and measures the degree of choice in a game knowing that the optimum is not necessarily an outcome of the game. We provide upper and lower bounds for these measures in different games.

As long as games does not generally yield outcome as expected, it is necessary to design a mechanism that interacts with players so that their self-interested behaviors lead to a desirable outcome. We are interested in coordination mechanisms and online mechanism designs in which the underlying data is unknown. Moreover, in online mechanism design, the decision must be made without knowledge of future information in the sense of online algorithms. We give an optimal coordination mechanism for scheduling games and optimal online mechanisms for an online auction problem where the auctioneer needs to efficiently allocates perishable items and compute the payment for single-minded bidders.