

Homework 4

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1 Primal-Dual Formulation

Formulate and write the relaxed linear programs together with their dual for the following problems. For the last problem (Shortest Super String), no dual is required. Remember to define the variables in the primal. For example, in Vertex Cover, we defined for $u \in V$:

$$x_u = \begin{cases} 1 & \text{if } u \text{ is in the vertex cover,} \\ 0 & \text{otherwise.} \end{cases}$$

Set Cover Given an universe $U = u_1, \dots, u_n$, a collection of subset $\mathcal{S} = \{S_1, \dots, S_k\}$, and a cost function $c : \mathcal{S} \rightarrow \mathbb{Q}^+$. A *set cover* is a sub-collection \mathcal{C} of \mathcal{S} that covers all element in U , i.e., for all $u \in U$, $u \in \bigcup_{S \in \mathcal{C}} S$. Find a minimum cost set cover.

Minimum Spanning Tree Given an undirected graph $G(V, E)$ and a weight function $w : E \rightarrow \mathbb{Q}^+$. Find a minimum weight spanning tree, i.e., a tree spans all vertices V .

Steiner Tree Given an undirected graph $G(V, E)$, a weight function $w : E \rightarrow \mathbb{Q}^+$ and a set of terminal $S \subset V$. Find a minimum weight tree that spans all node in S .

Traveling Salesman tour Given a complete undirected graph $G(V, E)$, a weight function $w : E \rightarrow \mathbb{Q}^+$. Find a minimum weight simple cycle that visits all nodes in V .

Shortest Super String Given a set of n strings $\{s_1, \dots, s_n\}$ over a finite alphabet Σ . Find a minimum length string t that contains each s_i as a substring.

2 Duality

Solve exercise 12.8 in the *Approximation Algorithms*.