Approximation Algorithms	**Dept. of CS, Aarhus University**
Homework 3	
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Load Balancing

Consider the problem LOAD BALANCING (or MINIMUM MAKESPAN SCHEDULING): Given n jobs $J_1, \ldots, J_n$ with processing time $p_1, \ldots, p_n$, respectively and m identical machines. Find a schedule of these jobs such that the makespan is minimized.

Question 1

Consider the following greedy algorithm.

Algorithm 1 A greedy algorithm for LOAD BALANCING

1: Order the jobs arbitrarily.
2: **for** $i = 1$ to n **do**
3: Assign J_i to the machine with least current load.
4: Update the load.
5: **end for**
6: **return** the assignment.

1. Prove that the greedy algorithm gives $\left(2 - \frac{1}{m}\right)$-approximation and give an tight example for the algorithm.
2. Take a modification on the first step: order the jobs in decreasing order of processing time. Prove that the new algorithm gives 3/2-appoximation

Hint: Let L be the makespan and let M_i be a machine with load L (in the output). Consider the last job assign to machine M_i.

Question 2

Assume that there are k distinct processing times, i.e., $p_i \in \{a_1, \ldots, a_k\}$ where k is a constant. Given $T > 0$. Design a dynamic programming whose running time is $O(n^{2k})$ such that:

- Return **No** if the minimum makespan is larger than T,
- Return **Yes** and a schedule that has makespan smaller than T otherwise.

Hint: Define $M(x_1, \ldots, x_k) :=$ the number of machines needed to schedule x_i jobs of processing time a_i (for $1 \leq i \leq k$) such that the makespan is smaller than T.