Fundamental Computer Science
Turing Machines (extensions)
Training session

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Consider a set \( A = \{a_1, a_2, \ldots, a_n\} \) of positive integers and an integer \( w \in \mathbb{N} \).

Give a Non-deterministic Turing Machine that recognizes the language \( L = \{A' \subseteq A : \sum_{a_i \in A'} a_i = w\} \).
1. choose non-deterministically a set $A' \subseteq A$
2. add the elements of $A'$
3. if they sum up to $w$, then accept
Solution

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2. add the elements of $A'$
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- How to choose $A'$ non-deterministically?
  - produce all binary numbers of $n$ digits
  - start from $00\ldots0$ and add 1 at each iteration
Write a program for a Random Access Turing Machine that multiplies two integers. Assume that the initial configuration is $(1; 0, a_1, a_2, 0; \emptyset)$.
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1: while \(R_1 > 0\) do
2: \(R_1 \leftarrow R_1 - 1\)
3: \(R_3 \leftarrow R_3 + R_2\)
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or (all computations should pass through \(R_0\))

1: \quad R_0 \leftarrow R_1
2: \quad \textbf{while } R_0 > 0 \textbf{ do}
3: \quad R_0 \leftarrow R_0 - 1
4: \quad R_1 \leftarrow R_0
5: \quad R_0 \leftarrow R_3
6: \quad R_0 \leftarrow R_0 + R_2
7: \quad R_3 \leftarrow R_3
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1: while $R_1 > 0$ do
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or (all computations should pass through $R_0$)

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2: while $R_0 > 0$ do
3:    $R_0 \leftarrow R_0 - 1$
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6:    $R_0 \leftarrow R_0 + R_2$
7:    $R_3 \leftarrow R_3$

1: load 1
2: jzero 9
3: sub =1
4: store 1
5: load 3
6: add 2
7: store 3
8: jump 1
9: halt