Proof of Zeckendorf's Theorem

Objective: Study the Fibonacci numbers as a *numbering system*.

Let us first introduce a notation: $j \gg k$ iff $j \geq k + 2$.

Zeckendorf's theorem

every positive integer n has a unique representation of the form:
$n = F_{k_1} + F_{k_2} + ... + F_{k_r}$ where $k_1 \gg k_2 \gg ... \gg k_r$ and $k_r \geq 2$

- Here, we assume that the Fibonacci sequence starts at index 1 and not 0, moreover, the decompositions will never consider $F(1)$ (since $F(1) = F(2)$).

Use the Theorem as a numbering system

Any unique system of representation is a numbering system.

- The previous theorem ensures that any non-negative integer can be written as a sequence of bits b_i
- Detail the algorithm that adds a 1 to an integer (increment).