Maths for Computer Science
Correction of the Quizz – Part 2

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Dealing with infinity

- A simple question with multiple answers
- A nice puzzle
- Going further: The Hilbert's hotel

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Message

We can not perform classical arithmetic operations at infinity!

From finite summations to infinite

- Dealing with limits

Asymptotic notations

- upper bound
  $f = O(g) \Leftrightarrow \exists C > 0\ \exists n_0 > 0\ \forall n > n_0\ f(n) \leq Cg(n)$
- lower bound
  $f = \Omega(g) \Leftrightarrow g = O(f)$
- $f = \Theta(g) \Leftrightarrow f = O(g)$ and $f = \Omega(g)$

Irrational numbers

Example

Computing $\sqrt{2}$

- It can not be written as the ratio of two integers.
- Negative results are usually harder to establish than positive (constructive) ones.

Functions

Definition

a function from a set S to a set T is a rule that assigns a unique value from T to every value from S.

- This notion is more restrictive than necessary. Think, e.g., of the operation *division* on integers.
  - We learned at school that division is a function that assigns a number to a given pair of numbers—yet we are warned immediately not to "divide by 0" since the quotient upon division by 0 is "undefined",
  - So, division is *not quite* a function of the same sort as addition or multiplication, which both *do* conform to the notion envisioned by the classical definition.

Injective functions

F is injective if or each $t \in T$, there is at most one $s \in S$ such that $F(s) = t$

- “multiplication by 2” is injective: If you are given an even integer $2n$, you can always respond with the integer n.
- “integer division by 2” is not injective because performing the operation on arguments $2n$ and $2n + 1$ yields the same answer (namely, n).

Binary and hexadecimal

2021

- $2021 = 1024 + 512 + 256 + 128 + 64 + 32 + 8 + 1$
  Thus, the binary writing is: $(11111101001)_2$
- $2021 = 7 \times 256 + 229$ and $229 = 14 \times 16 + 5$
  thus, $(7, 14, 5)_{16}$

Equivalence relations

A binary relation ρ is an equivalence relation if the three following properties are fulfilled:

1. Reflexivity: $x \rho x$
2. Symmetry: if $x \rho y$ then $y \rho x$
3. Transitivity: if $x \rho y$ and $y \rho z$ then $x \rho z$

Training

Prove that the following relation between pairs of integers (n_i, m_i):
$(n_1, m_1)\rho(n_2, m_2)$ iff $n_1 + m_2 = n_2 + m_1$ is an equivalence relation

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- Intuitively, this relation reflects the geometrical argument that states that the two pairs of points (n_1, m_1) and (n_2, m_2) are equivalent iff the differences $n_1 - m_1$ and $n_2 - m_2$ are equal.
- Draw the picture.
- Thus, the equivalence classes here correspond to straight lines parallel to the first bisectrice and in particular the line that contains the point $(1, 0)$.

Logarithms

Definition

b is the base.
$log(x)$ is defined as the inverse of the exponentiation $f(x) = b^x$:
$x = b^{log_b(x)}$

Using this definition and the basic property of the exponential, we can establish all the properties of the log.

- $log_b(x \cdot y) = log_b(x) + log_b(y)$
- $log_b(1) = 0$ is a consequence of this definition, not by convention![1]
- $\log_a(x) = \log_a(b) \log_b(x)$

[1]Easy to prove

Another expression of $n^{\log_a(b)}$

- $n^{\log_a(b)} = b^{\log_a(n)}$