Fundamental Computer Science

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(inspired by Giorgio Lucarelli)

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Random Access Turing Machines

- Random Access Memory
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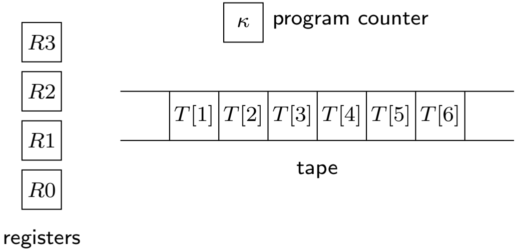
  - access any position of the tape in a single step
- we also need:
  - finite number of *registers* $\rightarrow$ manipulate addresses of the tape
  - *program counter* $\rightarrow$ current instruction to execute



- program: a set of instructions

Random Access Turing Machines: Instructions set

instruction	**operand**	**semantics**
read	j	$R_0 \leftarrow T[R_j]$
write	j	$T[R_j] \leftarrow R_0$
store	j	$R_j \leftarrow R_0$
load	j	$R_0 \leftarrow R_j$
load	$= c$	$R_0 = c$
add	j	$R_0 \leftarrow R_0 + R_j$
add	$= c$	$R_0 \leftarrow R_0 + c$
sub	j	$R_0 \leftarrow \max\{R_0 + R_j, 0\}$
sub	$= c$	$R_0 \leftarrow \max\{R_0 + c, 0\}$
half		$R_0 \leftarrow \lfloor \frac{R_0}{2} \rfloor$
jump	s	$\kappa \leftarrow s$
jpos	s	if $R_0 > 0$ then $\kappa \leftarrow s$
jzero	s	if $R_0 = 0$ then $\kappa \leftarrow s$
halt		$\kappa = 0$

- register R_0: *accumulator*

Random Access Turing Machines: Formal definition

A Random Access Turing Machine is a pair $M = (k, \Pi)$, where

- $k > 0$ is the finite number of registers, and
- $\Pi = (\pi_1, \pi_2, \ldots, \pi_p)$ is a finite sequence of instructions (program).

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Notations

- the last instruction π_p is always a *halt* instruction
- $(\kappa; R_0, R_1, \ldots, R_{k-1}; T)$: a **configuration**, where
  - κ: program counter
  - R_j, $0 \leq j < k$: the current value of register j
  - T: the contents of the tape
    (each $T[j]$ contains a non-negative integer, i.e. $T[j] \in \mathbb{N}$)
- **halted configuration:** $\kappa = 0$

Exercise

- Write a program for a Random Access Turing Machine that multiplies two integers.
  Tip: assume that the initial configuration is $(1; 0, a_1, a_2, 0; \emptyset)$

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```
1: load 1
2: jzero 9
3: sub =1
4: store 1
5: load 3
6: add 2
7: store 3
8: jump 1
9: halt
```