
TRAINING ON DIVISIBILITY

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This example is a generalization of the well-known problems of filling jugs.

Let us study a scene of the B-grade american action movie *Die Hard 3 – a vengeance*. Samuel Jackson and Bruce Willis have to disarm a bomb placed by the diabolic Simon Gruber. There are two empty jugs (one of 3 gallons and the other with 5 gallons) in front of a water fountain. Disarming the bomb requires to isolate precisely 4 gallons of water and place it on the scale to stop the timer...

Let us consider the general problem with two jugs of capacity a and b (without loss of generality, let consider $a \leq b$). The quantity to isolate is denoted by c .

- **Question 0.** Solve the problem by hand as Bruce Willis did (for $a = 3$, $b = 5$ and $c = 4$).
- **Question 1.** Write the first steps of the process and characterize the possible moves. Determine an invariant property¹.
- **Question 2.** Derive a solution for the instance: $a = 3$, $b = 6$ and $c = 4$. What could we say about the instance: $a = 21$, $b = 26$ and $c = 3$?
- **Question 3.** Prove that the amount of water in each jug is always a multiple of $\text{GCD}(a,b)$.
- **Question 4.** Derive a greedy algorithm for solving the problem².

¹Hint: the amount of water in each jug is always a linear combination of a and b

²Hint: write c as the smallest possible linear combination of a and b