The following expression gives a fast way to compute the term of the Fibonacci series.

Proposition 1.
$F(2n) = F(n)^2 + F(n-1)^2$
$F(2n+1) = (2.F(n-1) + F(n)).F(n)$

Prove the two following propositions on Lucas' Numbers.

Proposition 2.
$F(n+1) = \frac{1}{2}(F(1).L(n) + F(n).L(1))$

The previous property can be extended for any $m > 1$ as follows:

Proposition 3.
$2.F(n+m) = F(m).L(n) + F(n).L(m)$

Zenckendorf's Theorem and its use.

How Fibonacci numbers can be used for representing integers?

Let us first introduce a notation: $j \gg k$ iff $j \geq k + 2$.
We will first prove the *Zeckendorf's theorem* which states that every positive integer n has a unique representation of the form:
$n = F_{k_1} + F_{k_2} + ... + F_{k_r}$ where $k_1 \gg k_2 \gg ... \gg k_r$ and $k_r \geq 2$.
Here, we assume that the Fibonacci sequence starts at index 1 and not 0, moreover, the decompositions will never consider $F(1)$ (since $F(1) = F(2)$).

Use as a numbering system:
Any unique system of representation is a numbering system.
The previous theorem ensures that any non-negative integer can be written as a sequence of bits b_i