# Rules

The work has to be done by group of 2 to 3 people.

The work should be sent before thursday midnight by email to Denis.Trystram@univ-grenoble-alpes.fr
The naming convention are:

- Subject: MOSIG1 Fibo numbering
- Attached file: FiboNumberingYourname

Indicate clearly your sources
provide details on the repartition of the work within the group (ideas, technical proof, writing, other)
give an estimation of the time you took for solving the problem

# Part 1. Proof of Zeckendorf's Theorem

**Objective:** Study the Fibonacci numbers as a numbering system.

Let us first introduce a notation: $j \gg k$ iff $j \geq k + 2$.
Prove the *Zeckendorf's theorem* which states that:

every positive integer $n$ has a unique representation of the form:
$n = F_{k_1} + F_{k_2} + ... + F_{k_r}$ where $k_1 \gg k_2 \gg ... \gg k_r$ and $k_r \geq 2$

Here, we assume that the Fibonacci sequence starts at index 1 and not 0, moreover, the decompositions will never consider $F(1)$ (since $F(1) = F(2)$).

# Part 2. Use the Theorem as a numbering system

Any unique system of representation is a numbering system.
The previous theorem ensures that any non-negative integer can be written as a sequence of bits $b_i$

Detail the algorithm that adds a 1 to an integer (increment).