UNIVERSITÉ Grenoble Alpes

Master of Science in **Informatics** at **Grenoble**

Grenoble INP ensimag

UE Mathematics for Computer Science

Exercises in combinatorics

Choosing a team

You want to choose a team of m people from a pool of n people for your startup company, and from thesempeople you want to choose k to be the team managers. You took the *Mathematics for Computer Science course*, so you know you can do this in

$$\binom{n}{m}\binom{m}{k}$$

ways. But your manager, who went to Harvard Business School, comes up with the formula

$$\binom{n}{k}\binom{n-k}{m-k}$$

Before doing the reasonable thing, dump on your manager, you decide to check his answer against yours.

1. Start by giving an algebraic proof that your manager's formula agrees with yours.

2. Now give a combinatorial argument proving this same fact.

A curious decomposition

Now try the following, more interesting theorem:

$$n2^{n-1} = \sum_{k=0}^{n} k\binom{n}{k}$$

1. Start with a combinatorial argument. Hint: let $\mathcal{S}$ be the set of all sequences in $\{0, 1, \star\}^n$ containing exactly one $\star$.

2. How would you prove it algebraically?

Covering

Let $\mathcal{E}$ a set of n elements. A 2-covering is a couple subsets (A, B) of $\mathcal{E}$ such that $A \cup B = \mathcal{E}$. Compute the number of 2-covering.

No adjacency

There are 20 books arranged in a row on a shelf.

1. Describe a bijection between ways of choosing 6 of these books so that no two adjacent books are selected and 15bit sequences with exactly 6 ones.

2. How many ways are there to select 6 books so that no two adjacent books are selected?

Combinatorial identity

Prove the following theorem

$$\sum_{i=0}^{n} \binom{k+i}{k} = \binom{k+n+1}{k+1}$$

1. using a combinatorial argument;

2. using induction.