UFR IM²AG — Université Grenoble Alpes

Master of Science in Informatics at Grenoble

Grenoble INP ensimag

UE Mathematics for Computer Science

Homework, November 2020

Certify on your homework:
I understand what plagiarism entails and I declare that this report is my own, original work.
Name, date and signature.

- The firm deadline is November Wednesday 25 8:00 am (before my lecture).
- The homework should be 4 pages (and appendix)
  in the pdf format (scanned manuscripts in pdf are allowed)
- the filename should be FamilyName-Mosig-MfCS-HW1.pdf
- send with **your official mail** at Jean-Marc.Vincent@univ-grenoble-alpes.fr with the subject `[MOSIG1:MfCS] HW1 FamilyName`

Context

A Computer Programming Contest is organized with students in L3. A set of problems is proposed and teams of students solve the problems and submit the solutions on an on-line judge. A ranking is established at the end of the contest and a price is delivered to the best teams, then the price is fairly splitted among the students of the winning team.

At the beginning students compose teams as they want, there are no restrictions on the number of students in a team. A student could do it solo or we could have all the students in a single team. Of course a student cannot belong to two different teams. A configuration of the students is the set of all teams (each student is in only one team).

As a typical example, the set of students is $\{s_1, s_2, \cdots, s_{13}\}$ and a particular set of teams

$Team$	$Students$
A	s_2, s_4, s_{10}
B	s_3
C	s_1
D	$s_5, s_6, s_7, s_8, s_9, s_{13}$
E	s_{11}, s_{12}

In this specific case, we have 5 teams, 2 solo teams, the largest team has 6 students, the decomposition is $\{s_2, s_4, s_{10}\}, \{s_3\}, \{s_1\}, \{s_5, s_6, s_7, s_8, s_9, s_{13}\}, \{s_{11}, s_{12}\}$.

We suppose that all configurations are equally probable and we want to estimate some parameters to calibrate the environment of the contest (number of rooms, material, size of teams,...)

The problem is to evaluate properties of random decomposition in teams, the approach is to combine counting techniques, simulation and basic statistics. The choice of the methods must be justified clearly.

1. **Number of teams** Evaluate the number of teams decomposition, (the number of involved students is around 100)
2. **Simulation Algorithm**
   (a) Propose an algorithm that generates uniformly a team decomposition on $\{1, \cdots, n\}$.
   (b) Justify this algorithm and prove it.
3. **Average Properties** For a uniformly random decomposition on $\{1, \cdots, n\}$ compute the following quantities, for each computation you can either compute algebraically (if possible) or simulate and make some statistics.
   (a) Compute the average number of teams
   (b) Compute the probability to have more than k teams
   (c) Compute the average number of solo teams
   (d) Compute the distribution (histogram) of the size of the largest team
4. **Synthesis**
   Make a synthesis on the properties of uniform random decomposition in disjoints teams.

Comments

- You are free to choose your programming language.
- The size of the set of students is a parameter of the problem, you have to choose different values to provide interesting conclusions.
- For the simulations, take care of the size of the samples and provide confidence intervals on the estimations of averages.
- You can use appendix if you wish, but the 4 mandatory pages should be self-contained and are to be evaluated.