Coding

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Lecture notes **Maths for Computer Science** – *2019*

Gray Code

This code was invented by a french mathematician of the XIXth century (Louis Gros in 1872), reinvented by Franck Gray at Bell's lab in 1930.

There exist several variants of Gray codes. Let us present the most popular one, namely the *reflected Gray code* whose principle is depicted in Figure 1. The 1-bit Gray code is simply 0 and 1. The next one (for 2-bits) is obtained by mirroring the 1-bit code and prefix it by 0 and 1. The next ones are obtained similarly (see Figure 1).

The most important characteristic is the coding from one position to the next is to flip only one bit. There is a simple way to determine the bit that changes from a position to the next one:

if the number of bit at 1 in position i is even, then flip the last bit, otherwise, flip the bit left to the rightmost bit equal to 1.

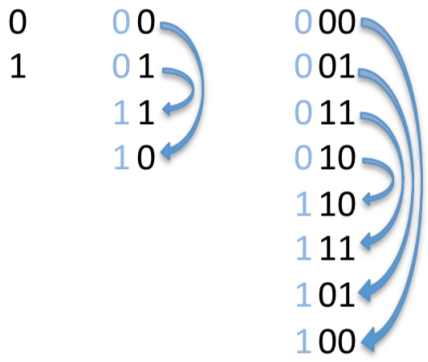

Figure 1: Construction of reflected Gray codes ($n = 1, 2, 3$).

Gray code can easily be determined from the classical binary representation as follows (see Figure 2):

$(x_{n-1}x_{n-2}...x_1x_0)_2$

shift right: $(0x_{n-1}x_{n-2}...x_1)_2$

Take the exclusive OR (bit-to-bit) between the binary code and its shifted number:

$(x_{n-1}(x_{n-2} \oplus x_{n-1})...(x_0 \oplus x_1))_G$.

For instance in the example of the figure, the binary code of $5 = (00101)_2$ is $(0 \oplus 0)(0 \oplus 0)(0 \oplus 1)(1 \oplus 0)(0 \oplus 1) = (00111)_G$.

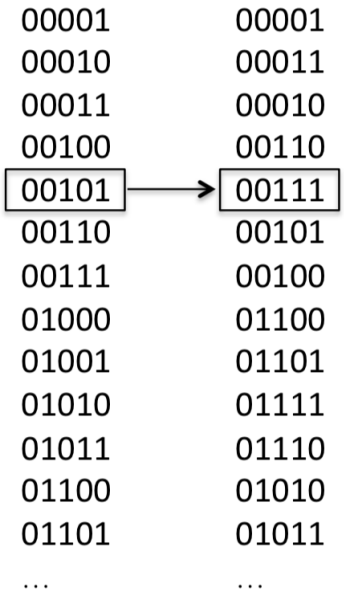

Figure 2: From binary to reflected Gray code.