Variations on Hanoi's Towers

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Maths for Computer Science – *Home Work MOSIG 1 – 2018*

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The purpose of this work is to study the mathematical properties of several variants of the classical puzzle *Towers of Hanoi*.

1 Definition of the basic (classical) Hanoi Towers

The puzzle has been proposed by the french mathematician Edouard Lucas in the late IXX-th century. It consists in moving a pile of n disks from one peg (the Departure) to the Arrival peg using the Intermediate one.

More precisely, we are looking for the minimum number of moves, according to the following rule: we can not put a disk on the top of another one whose diameter is larger, see figure 1.

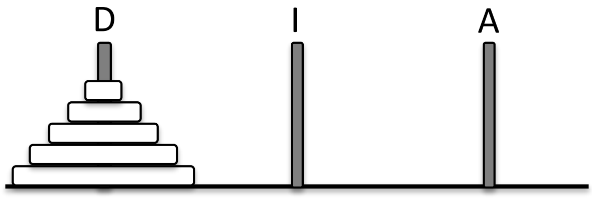

Figure 1: Initial position of the puzzle (for $n = 5$ disks).

Analysis

Recall the principle for solving this puzzle, and perform its cost analysis.

2 Coding of the position

What is the minimum number of bits for coding a position?
For coding a move?

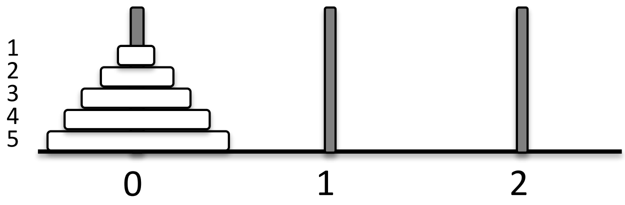

Figure 2: Coding scheme.

3 First variant: unbounded numbers of pegs

Let consider now the same problem with more pegs (denoted by k). Give an algorithm which achieves the minimum number of possible moves and provide the corresponding cost analysis.

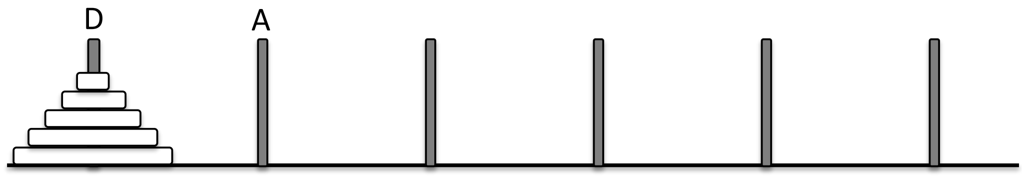

Figure 3: Initial state for $n = 5$ and $k = 6$.

4 Improved solutions

Let $H(n, k)$ be the number of moves for n disks and k pegs.

According to the previous results, we have two bounds: $H(n, \Theta(1)) = \Theta(2^n)$ and $H(n, \Theta(n)) = \Theta(n)$.

We are interested here on the problem of determining $H(n, k*)$, that is the best number of moves (the smallest) to keep a linear cost.
In particular, can we keep a linear cost for instances like the one of figure 4?

The last question is more open. It is to give (and analyze) several intermediate strategies which achieve a trade-off between the number of pegs and the cost.

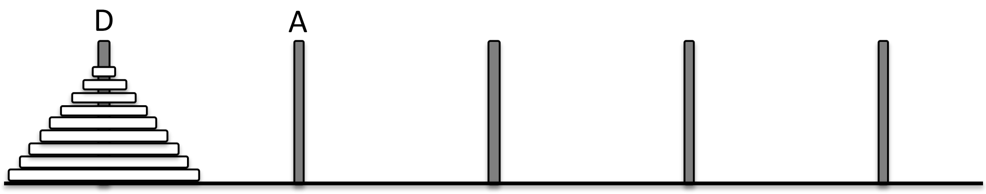

Figure 4: Instance with $n = 9$ disks and $k = 5$ pegs.