UNIVERSITÉ Grenoble Alpes — Master of Science in Informatics at Grenoble — Grenoble INP ensimag

UE Mathematics for Computer Science

Homework, December 2018

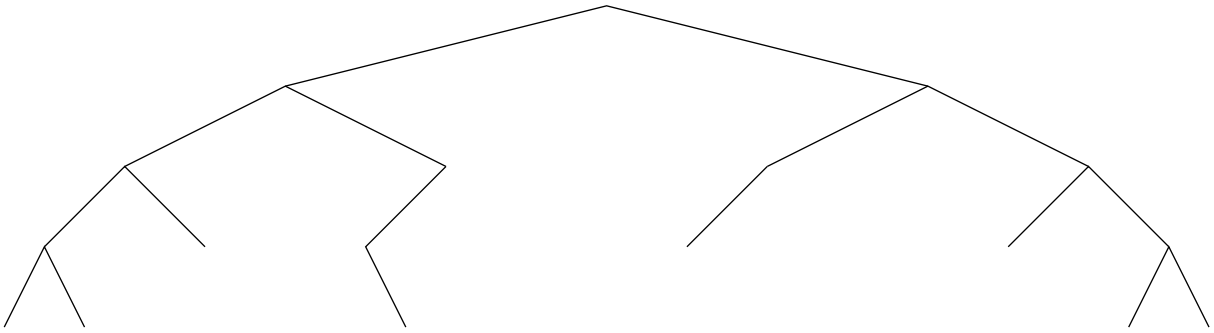
Certify on your homework:
I understand what plagiarism entails and I declare that this report is my own, original work.
Names, date and signatures.

- The firm deadline is December Friday 21 midnight (before Saturday).
- The homework should be 4 pages (and appendix)
  in the pdf format (scanned manuscripts in pdf are allowed)
- the filename should be FamilyName1-FamilyName2-Mosig-MfCS-HW2.pdf
- send with your official mail at Jean-Marc.Vincent@univ-grenoble-alpes.fr with the subject

  `[MOSIG1:MfCS] HW2 FamilyName1 FamilyName2`

Binary Trees

The problem is to evaluate properties of random binary trees, the approach is to combine counting techniques, simulation and basic statistics. In the problem we consider shape of trees, where nodes are defined by their position regarding the root of the tree. (Each node have a left subtree or a right subtree that sometimes are empty).



$n = 18 \qquad h = 4 \qquad h^{av} = \frac{1}{18}(0 \times 1 + 1 \times 2 + 2 \times 4 + 3 \times 6 + 4 \times 5) = \frac{48}{18} \simeq 2.66$

$l = 8$

1. **Simulation Algorithm**
    1. Propose an algorithm that generates uniformly a random binary tree with n nodes.
    2. Justify this algorithm and prove it.
2. **Statistical Properties** For a uniformly generated binary tree with n nodes compute the following quantities, for each computation you can either simulate and/or make some statistics or compute it directly.
    1. The average of L_n number of leaves and the probability distribution of L_n the number of leaves.
    2. The average maximum height H_n of the tree and the probability distribution of the height H_n.
    3. The average height H_n^{av} of the tree and the probability distribution of the height (the height of a node is its distance to the root, the average height of a tree is the average of the height of each node)..
3. **Synthesis**

    Make a synthesis on the properties of uniform random binary trees.

Comments

- You are free to choose your programming language.
- The number of nodes n is a parameter of the problem, you have to choose different values for n to provide interesting conclusions.
- For the simulations, take care of the size of the samples and provide confidence intervals on the estimations of averages.
- You can use appendix if you wish, but the 4 mandatory pages should be self-contained and are to be evaluated.