UE Mathematics for Computer Science

Exercises in combinatorics

Name 1: Name 2:
Name 3: Name 4:
indicate the writer by a $\star$

Let $S_n^k = 1^k + 2^k + \cdots + n^k$. This sums have been studied in the previous part of this course in the case $k = 1$, $S_n^1 = \Delta_n$, the triangular numbers and the case $k = 2$, $S_n^2 = \Theta_n$. The aim of this exercise is to find a computation method for the S_n^k.

Combinatorial Identity

Prove by a double counting argument that $\displaystyle\sum_{k=p}^{n} \binom{k}{p} = \binom{n+1}{p+1}$. (1)

Explain why we could write the identity starting from 0 $\displaystyle\sum_{k=0}^{n} \binom{k}{p} = \binom{n+1}{p+1}$ (2)

First steps

Compute $\displaystyle\sum_{k=0}^{n} \binom{k}{1}$ and check your formula using Δ_n.

Then with the same approach compute $\displaystyle\sum_{k=0}^{n} \binom{k}{2}$ and check your formula using Δ_n and Θ_n.

Generalization

Propose a method to compute S_n^k and apply it to compute S_n^3.